PERSONAL TAXES AND THE
FAIR RATE OF RETURN DOCTRINE

Dan Palmon* and Uzi Yaari**

In setting the allowable rate of return on public utilities' asset base, commissioners and courts in recent years have continued to rely heavily on the standard discounted cash flow (DCF) method, based on the Gordon-Miller-Modigliani model of share valuation under constant growth

\[ p = \frac{E (1 - eb)}{r - \bar{g}} \]

where \( E \) is year-end earnings per share after corporate tax and \( \bar{g} \) their annual growth rate; \( b \), annual investment expressed as a fraction of the firm's total earnings; and \( e \), the fraction of this investment financed by retention. Due to the assumption of no personal taxation, the firm's cutoff rate of return on equity-financed incremental investment, \( \rho \), is not distinguishable from the shareholders' required post-tax rate of return, \( r \). This rate is derived from the price equation and empirically evaluated by

\[ \rho = r = \frac{E (1 - eb)}{p} + \bar{g} \]

The standard formulation of the firm's equity investment cutoff rate and the underlying valuation model have been refined by academic contributions in the past decade to accommodate a number of realistic features, including the effect of personal taxation [1, 6, 8, 10, 11]. According to a consensus among financial theorists, personal taxes are viewed as a wedge between the return earned by the firm and its shareholders. If the latter earn their return in the form of cash dividends subject to tax at the rate \( \delta \), and new projects are financed exclusively by issuing stock (paid for with post-dividend-tax funds), the rate of return on new projects must be at least \( r/(1 - \delta) \). In contrast, if the firm finances new projects exclusively by retaining earnings and in the process subjects its shareholders to capital gains tax at the rate \( \gamma \), the yield of those projects should be at least \( r/(1 - \gamma) \). More generally, if the firm relies on a given ratio of internal/external equity financing, the investment cutoff rate becomes a weighted average of the rates associated with the two extreme financial policies (formal derivation is provided below), namely

\[ \rho = e \frac{r}{1 - \gamma} + (1 - e) \frac{r}{1 - \delta} \]

Despite its theoretical and intuitive appeal, this formulation of the cutoff rate has brought about no change in the standard DCF method as used by

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regulatory agencies in determining the allowable rate of return of public utilities. Furthermore, since rate of return hearings offer a receptive audience for theoretical discourse, it is probable that the revised method has been ignored by corporate financial management in general.

There are a few possible explanations for the apparent appeal of the standard method to financial practitioners.

One explanation lies in the argument that the relevant effective tax rates are so low that personal taxation may be safely ignored. In this vein, Miller and Scholes [9] elaborate on the potential importance of the time-honored tax dodge of financing a tax-favored investment such as common stocks by borrowing, while offsetting the resulting interest expenses against income from that investment. Although the theoretical existence of this and similar devices of tax arbitrage is widely recognized, their alleged effectiveness can be disputed on empirical grounds. Specifically, there is evidence that the effective marginal tax rate of income from this source is considerable and close to the nominal rate. For example, Elton and Gruber [2] and Kalay [7] show that — consistent with a substantial difference between the effective marginal tax rates of dividends and capital gains — stock prices typically drop on ex-dividend day only by a fraction of the amount of dividends paid. For the period April 1966-March 1967, both studies indicate that the marginal tax rate on dividends paid in the NYSE averaged close to 40 percent. A similar figure is arrived at by Feldstein and Frisch [4] in a direct estimate based on individual tax returns in 1973.

A related explanation for the limited practical interest in the revised DCF method lies in the difficulty of estimating the marginal tax bracket of shareholders. This consideration seems particularly relevant in the case of public utilities, where estimates must be defended in court. Contrary to this argument, it is noted that a similar type of empirical judgement is routinely exercised under the standard DCF method, for instance, in estimating the anticipated growth rate based on past growth.

A third explanation for continued reliance on the simpler of the two methods lies in the seeming resemblance between them. In focusing attention on a critical rate of return at the level of the firm rather than of its shareholders, corporate and regulatory decision makers may have presumed similarity between the cutoff rate derived from the tax-free model and the pre-tax cutoff rate derived from a model which does take personal taxes into account.

Responding to the third explanation, this paper examines the difference between the two empirical methods. Findings indicate superiority of the revised version by showing that the apparent exclusion of personal taxes under the standard method implies their inclusion in a distorted way: taxes enter equation (1) indirectly by their effect on the current price. The resulting estimated cutoff rate is likely to contain a substantial bias which capriciously varies in size with the firm’s retention and growth rates, the marginal tax bracket of shareholders, and the extent of tax exemption accorded capital gains. The
analysis reveals that assessment based on the standard formula may considerably understate the cutoff rate under external equity financing and overstate it under internal financing, causing the allowable overall rate of return to fail the standard of parity set by the Hope doctrine [12]. It may also have a distortive side effect by inducing utilities to rely excessively on internal financing at the expense of external financing and to change the level of debt.

The plan of the paper is to derive, step-by-step, an empirical formula for the revised cutoff rate, expressed theoretically by (2), which then can be compared with the standard formula given by (1). This is achieved in the remaining sections as follows. First, empirical formulation of r is derived based on dividend valuation. Second, dividend and earnings valuation formulas are used to express share growth as a function of the firm’s growth rate — a step toward stating price as a function of the yield on incremental investment. Third, the formal expression of the revised cutoff rate stated in (2) is confirmed by differentiating price with respect to investment yield. Empirical formulation of the revised cutoff rate is derived from its formal expression and the empirical formulation of r and then compared with the cutoff rate derived by the standard method. Fourth, numerical examples with plausible parameters illustrate the potential sign and size of bias associated with the standard method. Fifth and last, implications of the findings for utility rate regulation are briefly stated.

DIVIDEND VALUATION AND EMPirical FORMULATION OF $r$

As before, let $\delta$ denote the marginal dividend tax rate and $\gamma$ the marginal rate of capital gains tax, where gains are assumed to be realized at the end of each year following distribution, reinvestment, and issuance of stock.

Based on the dividend valuation approach, ownership of a single share entails the following perpetual cash flows

<table>
<thead>
<tr>
<th>Year</th>
<th>Post-tax Dividends</th>
<th>Capital Gains Tax</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$E (1 - eb) (1 - \delta)$</td>
<td>$-\gamma P \bar{g}$</td>
</tr>
<tr>
<td>2</td>
<td>$E (1 - eb) (1 - \delta) (1 + \bar{g})$</td>
<td>$-\gamma P \bar{g} (1 + \bar{g})$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$E (1 - eb) (1 - \delta) (1 + \bar{g})^{t-1}$</td>
<td>$-\gamma P \bar{g} (1 + \bar{g})^{t-1}$</td>
</tr>
</tbody>
</table>

The present value of these flows discounted at the post-tax rate $r$ is

$$P = \frac{E (1 - eb) (1 - \delta)}{r - \bar{g}} - P \frac{\gamma \bar{g}}{r - \bar{g}}$$

which yields the price formula

$$P = \frac{E (1 - eb) (1 - \delta)}{r - \bar{g} (1 - \gamma)}$$

(3)
solved for the shareholders' opportunity rate of return (cf. [6])

\[ r = \frac{E (1 - eb) (1 - \delta)}{P} + \frac{g (1 - \gamma)}{P} \]  

(4)

Since this rate is calculated on a post-tax basis, it is not surprising to find it lower than the tax-free rate stated by (1) for any given observable E, P, and \( g \). It is noteworthy, however, that in a world of tax the conventional rate represents a "pre-tax" rate of return maintaining no fixed relationship to the theoretically meaningful post-tax rate. This relationship varies arbitrarily across firms and over time, depending on the value of the endogenous variables \( b \) and \( e \), on exogenous growth opportunities, and on the tax parameters \( \delta \) and \( \gamma \).

**Expressing Observable \( \overline{g} \) in Terms of \( g \) and Other Variables**

The derivation of an empirical formulation of \( P \) utilizing the empirical formulation of \( r \) given in (4) requires that the growth rate of earnings per share, \( \overline{g} \), first be stated as an explicit function of earnings' overall growth rate \( g \). This is accomplished through comparison of (3), based on dividend per share (dividend approach), with an expression of price based on prorated dividends (earnings approach) derived next.

Cash flows associated with ownership of a fixed fraction of the firm's equity are

<table>
<thead>
<tr>
<th>Year</th>
<th>Post-tax Dividends</th>
<th>Capital Gains Tax</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( E (1 - b - \delta + eb\delta) )</td>
<td>(-\gamma P \overline{g})</td>
</tr>
<tr>
<td>2</td>
<td>( E (1 - b - \delta + eb\delta) (1 + g) )</td>
<td>(-\gamma P \overline{g}(1 + g))</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>( t )</td>
<td>( E (1 - b - \delta + eb\delta) (1 + g)^{t-1} )</td>
<td>(-\gamma P \overline{g}(1 + g)^{t-1})</td>
</tr>
<tr>
<td>...</td>
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<td>...</td>
</tr>
</tbody>
</table>

where growth of the share of earnings received by an original claim keeps pace with growth of total earnings at the rate \( g \). These perpetual flows yield the current value

\[ P = \frac{E (1 - b - \delta + eb\delta)}{r - g} - P \frac{\gamma \overline{g}}{r - g} \]

solved for \( P \),

\[ P = \frac{E (1 - b - \delta + eb\delta)}{r - g + \gamma \overline{g}} \]  

(5)
Market domination by rational investors ensures that share valuation based on the two approaches commands the same price, from which it follows that \( \bar{g} \) can be expressed as a function of \( g \) by setting (3) equal to (5),

\[
\frac{\bar{g}}{g} = \frac{g(1 - eb)(1 - \delta) - rb(1 - e)}{(1 - \gamma)(1 - b) - (\delta - \gamma)(1 - eb)}
\]

(6)

where \( \bar{g} \leq g \) for \( e \leq 1 \).

**FORMAL AND EMPIRICAL EXPRESSIONS OF \( \rho \)**

The formal expression stated in (2) showing the cutoff rate in terms of the shareholders’ opportunity rate of return, \( r \), is confirmed next. This is done under the standard simplifying assumption \( \partial \rho^* / \partial b = 0 \), implying that \( \rho^* \) is both the average and marginal rate of return on incremental investment. The minimum rate \( \rho \) that must be earned on investment financed by a given ratio of internal/external equity funds is calculated by differentiating the price as stated in (3) or (5) with respect to \( b \) (after expressing \( \bar{g} \) in terms of \( g \) as in (6), and substituting \( bp^* \) for \( g \)), then setting the derivative at zero and solving for \( \rho^* = \rho^5 \)

\[
\rho = e \frac{r}{1 - \gamma} + (1 - e) \frac{r}{1 - \delta} \quad (2')
\]

The empirical formulation of the cutoff rate under exclusively internal financing is obtained by substituting (4) for the unobservable \( r \) in (2') [or dividing (4) throughout by \( 1 - \gamma \)] after setting \( e = 1 \)

\[
\rho_{in} = \frac{E(1 - b)}{P} \cdot \frac{1 - \delta}{1 - \gamma} + \bar{g}
\]

Clearly, the relationship \( (1 - \delta) / (1 - \gamma) < 1 \) implied by \( \delta > \gamma \) indicates a cutoff rate lower than that derived by the standard method as stated in (1).

In a similar fashion, the empirical formulation of the cutoff rate under exclusively external financing is obtained by substituting (4) for \( r \) in (2') [or dividing (4) throughout by \( 1 - \delta \)] after setting \( e = 0 \).

\[
\rho_{ex} = \frac{E}{P} + \frac{1 - \gamma}{1 - \delta} \bar{g}
\]

The relationship \( (1 - \gamma) / (1 - \delta) > 1 \) indicates a rate unambiguously higher than that stated in (1).

These results show that the standard method overstates the cutoff rate when financing is internal and understates that rate when financing is external. The extent of this bias varies across firms and over time, depending upon firm rates of retention and growth, the shareholders’ marginal tax bracket,
and the extent of tax exemption accorded capital gains. The source of this bias is the arbitrary and partial manner in which personal taxes enter the standard formulation through their effect on price — and price alone.

**Numerical Illustration**

The sign and extent of this bias under various retention and growth rates are illustrated in Table 1, where hypothetical cutoff rates based on the standard and revised formulas are displayed side by side. Comparison of these rates reveals the following features. (a) The standard formula overstates the cutoff rate under internal financing and understates it under external financing; in both cases the bias may be considerable. Given $r = .10$, $\delta = .40$, $\gamma = .16$, $b = .30$, and $\rho^* = .24$, there is an upward bias of 1.89 percent under internal financing and a downward bias of 1.52 percent under external financing. (b) While the true cutoff rate is independent of the rates of reinvestment and growth, the rate indicated by the standard formula decreases in both, and substantially so. (c) It follows that under internal financing the (upward) bias of that formula decreases in the rates of retention and growth, while under external financing its (downward) bias increases in both rates. Under internal financing with $b = .30$, an increase of $\rho^*$ from .20 to .24 causes a decrease of the bias from 2.37 percent to 1.89 percent; given instead $\rho^* = .24$, an increase of $b$ from .30 to .36 causes a decrease of the bias from 1.89 percent to 1.31 percent. This pattern is reversed under external financing.

**Table 1**

**Biased Calculation of the Firm's Investment Cutoff Rate Under the Standard Method**

*(All numbers in percentage points; parameters used: $E = $1, $r = 10$ percent, $\delta = 40$ percent, $\gamma = 16$ percent)*

<table>
<thead>
<tr>
<th>Percentage Reinvested b</th>
<th>Rate of Return $\rho^*$</th>
<th>Internal Financing ($e = 1$)</th>
<th>External Financing ($e = 0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Standard $\dagger$</td>
<td>Revised</td>
<td>Bias</td>
</tr>
<tr>
<td></td>
<td>$\frac{E (1-b) - r}{P} + g$</td>
<td>$\frac{r}{1-\gamma}$</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>20</td>
<td>14.27</td>
<td>11.90</td>
</tr>
<tr>
<td></td>
<td>24</td>
<td>13.79</td>
<td>11.90</td>
</tr>
<tr>
<td>36</td>
<td>20</td>
<td>13.79</td>
<td>11.90</td>
</tr>
<tr>
<td></td>
<td>24</td>
<td>13.21</td>
<td>11.90</td>
</tr>
</tbody>
</table>

$\dagger$The growth rate of earnings and price per share $g$ is calculated from equation (6); under $e = 1$ this rate equals that of the entire firm, $g$. 
Implications

It is shown above that the standard tax-free DCF method yields a distorted measure of the investment cutoff rate, and that the source of this distortion is a partial indirect effect of personal taxes. In utility rate regulation this method allows an overall rate of return which is too low if the firm relies heavily on external equity financing, and too high if it leans toward internal financing. Paradoxically, these biases imply that regulated utilities are discouraged from competing for external equity funds and at the same time are allowed excessive return on internally generated funds — with an uncertain effect on the overall rate of return. The first bias is inconsistent with the legal principle affirmed in the Hope decision, that public utilities be allowed to maintain their competitive edge in the capital market; the second is inconsistent with the fundamental social objective of limiting the profits of a protected monopoly. Both biases are likely to have a secondary distortive effect in inducing utilities to seek higher return by shifting from external equity financing toward internal financing (and a different level of debt). These biases and their adverse effects are not avoided by the common method of averaging rate of return estimates across utilities of comparable risk if similar regulatory constraints tend to induce similar behavior among firms of the same industry. These effects are inherent in the standard tax-free DCF method and can be avoided by switching to the revised method which explicitly recognizes the impact of personal taxes.

Endnotes

1 At this point a distinction must be drawn between the term "cutoff rate" used throughout this paper and the common term "cost of capital." The first term refers to the required rate of return of projects financed by a given combination of fund sources; the second refers to the required return assuming that the optimal combination is used. The question of optimal financial policy is not dealt with here.

2 Nonconventional distribution methods are ignored. Opportunities for tax arbitrage which would enable shareholders to convert dividend income into capital gains also are ignored. Miller and Scholes [9] have argued that such opportunities exist, but the law facilitating this tax arbitrage was introduced only in 1975 and, as pointed out by Feenberg [3] and Feldstein and Green [5], even since that time could not have affected more than 3 percent of all dividend income, or one-tenth of 1 percent of all taxpayers.

3 The assumption of annual trading is non-restrictive since the tax rate could be adjusted to reflect the effect of deferred realization (see Yaari et al. [13]).

4 Transaction costs of stock flotation and dividend payment are ignored in order to focus on tax effects. These costs could be assimilated readily in the analysis.

5 Under the less restrictive assumption of \( \frac{\partial \rho^*}{\partial b} \neq 0 \), the minimum marginal rate \( \rho' \) is derived by totally differentiating \( P \) with respect to \( b \), setting the condition \( \frac{dP}{db} = \frac{dP}{\partial b} + (\frac{dP}{\partial \rho^*})(\frac{\partial \rho^*}{\partial b}) = 0 \) and solving for \( \rho' = \rho^* + b (\frac{\partial \rho^*}{\partial b}) \) at \( \rho^* = \rho \). The minimum marginal rate would be below the minimum average rate if \( \frac{\partial \rho^*}{\partial b} < 0 \).
The tax rate $\delta = .4$ chosen for illustration is consistent with estimates for 1966-1967 and 1973 reported earlier. The current rate is likely to be higher. The statutory tax rate of capital gains presently is set at $\gamma = .4\delta$, or $\gamma = .16$. This figure understates the current effective rate if $\delta > .4$ and overstates it if the actual marginal holding period is longer than one year.

References